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Solitons, Bose–Einstein Condensation, and Superfluidity in Helium II

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The analytic form of a wave propagating with a constant velocity and a permanent profile is inferrred for a weakly interacting Bose gas, using an exact (rather than asymptotic) solution of the field equation of the self-consistent Hartree model. The significance of this approach is indicated, especially when realistic interatomic potentials are used. In addition, the general relation between solitons and Bose–Einstein condensation is underlined by invoking the profound insight recently acquired in studies of the quantum liquids involved in the living state. It is concluded that solitons may occur in He II, and may play a significant role in the phenomenon of superfluidity.

1. INTRODUCTION

In spite of the considerable progress achieved in the last 50 years or so (Vinen, 1983), a full microscopic theory of He II has proved to be quite elusive. In particular, the question first raised by London (1938) regarding the relation between Bose–Einstein condensation and superfluidity is still as controversial as ever (March, 1983; Ghassib and Chester, 1984).

One of the reasons for this elusiveness is presumably the unwieldy character of this deceptively simple quantum liquid, which does not yield easily to the usual experimental probes. It seems reasonable, therefore, to look at other systems where Bose–Einstein processes may occur, but which are more amenable to these probes. In this manner we seek some indicators

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as to the real microscopic description of He II. This is attempted in both Table I and Section 2.

At the same time the possible existence of solitons has recently been speculated upon in both thin He II films (Biswas and Warke, 1980, 1983) and in the Madelung fluid (Nonnenmacher and Nonnenmacher, 1983). The picture that has emerged from solving the hydrodynamic equations corresponding to the nonlinear Schrödinger equation (NLSE) is as follows:

(i) Only purely attractive interactions yields soliton solutions. For purely repulsive interactions any initial data that vanish as $x \to \infty$ will eventually evolve into decaying oscillations (Nonnenmacher and Nonnenmacher, 1983).

(ii) The dominant asymptotic behavior of the solution of the NLSE, without the restriction to a purely attractive or purely repulsive interaction, has been obtained (Segur, 1976); this contains both solitons as well as decaying oscillations.

In this paper we consider the possible existence of solitary waves propagating in He II. We approach this problem for a system of weakly interacting bosons in a self-consistent field approximation (Gross, 1963), a

Table I. Physical Phenomena Involved in Various Manifestations of Quantum Liquid Behavior

Quantum fluid	Bose-Einstein condensate (0 K)	Solitons or solitary waves	Superfluidity
Ideal Bose gas	100%	No	No
Weakly interacting Bose gas	$\lesssim$100% (Gross, 1983)	Yes (this paper)	Yes (Reppy, 1984)
He II	~10% (Sears *et al.*, 1982)	Yes (this paper)	Yes (Kapitza, 1941)
Biosystems	Yes (Fröhlich, 1968)	Yes (Davydov and Kislukha, 1973)	Yes[a] (Fröhlich, 1977)
Superconductors	Yes (but see Leggett, 1980)	Yes (Tuszynski *et al.*, 1984)	Yes (Bardeen *et al.*, 1957)
Liquid ^{3}He	Yes (Leggett, 1975; but see Leggett, 1980)	Yes (Maki, 1978)	Yes (Osheroff *et al.*, 1972)

[a]Conjectured only.

long-tested model of superfluidity in He II, which has the double advantage of being mathematically simple and physically realizable in the form of liquid ^{4}He adsorbed on porous Vycor glass (Reppy, 1984). That the Hartree liquid model (Gross, 1963) may be relevant to real He II has already been demonstrated, provided the appropriate $U(1)$ gauge theory is considered as its natural extension (Chela-Flores, 1975; Chela-Flores *et al.*, 1977). More generally, by considering a hierarchy of increasingly refined effective Lagrangian densities (Ghassib and Chela-Flores, 1983; Ghassib and Khudeir, 1986; Chela-Flores and Ghassib, 1986), the range of validity of the Hartree model has been shown to be much wider than is commonly thought.

The remainder of this paper is as follows:

In Section 2 we consider the progress achieved in understanding some basic phenomena in the life sciences, where Bose–Einstein condensation produces order in momentum space, and where this order is propagated through the system by means of solitons. We argue that such knowledge has an important bearing on the general phenomenon of superfluidity. In Section 3 we discuss the self-consistent field approximation paying special attention to the choice of potential. In Section 4 we consider an exact solution to the field equation for a model, but nontrivial, potential that contains both attractive and repulsive components. We find that, for velocities exceeding a certain critical value, permanent waves propagate with a constant velocity. Finally, in Section 5, we reconsider the relationship between Bose–Einstein condensation and superfluidity within the present framework, concluding that solitons may act as agents for propagating the condensate throughout the medium. In passing, the intriguing question of the possible relation between solitons and dimers is discussed.

2. ON THE ANALOGY BETWEEN HELIUM II AND BIOSYSTEMS

Considerable progress has been achieved recently in the understanding of the living state by considering the novel perspective of order in momentum space. Two models have been proposed. The first hinges on condensation of phonons (Fröhlich, 1968) as well as on pairing of phonons (Chela-Flores, 1985). The second is based on soliton propagation in some proteins and nucleic acids (Davydov and Kislukha, 1973). These two seemingly different models of biological order have now been brought together (Tuszynski *et al.*, 1984b), with the corollary that Bose–Einstein condensation and soliton propagation are not just mathematically equivalent, but also physically complementary, in the sense that solitons are needed to propagate order in various types of biosystems. The underlying static effect (Bose–Einstein condensation) in these systems manifests itself in several ways:

1. Specific biological effects characterized by low frequencies (millimeter waves) have been found (Grundler and Keilman, 1983) in the temperature range 30–35°C, where the growth rate of yeast cells is appreciably altered.

2. It has been shown, using the Raman effect (Drissler, 1980) on algae, that certain chlorophyll lines in the range 100–3000 cm^{-1} are present at low temperatures, but disappear at room temperature.

3. Human red blood cells (erythrocytes) are only subject to Brownian motion until they get stuck together in rouleaux. These cells rush forward once they have approached each other to within 4 μm (Sewchand and Rowlands, 1983).

On the other hand, the dynamic effect (solitons) can also become evident:

1. It can aid in the understanding of some properties of the green alga *Chlorella pyrenoidosa* (Del Giudice *et al.*, 1981).

2. It can explain the interesting phenomenon of resonant absorption of microwave energy by aqueous solutions containing helical DNA of known length (Edwards *et al.*, 1984; Scott and Jensen, 1985).

3. It can explain certain phenomena related to the fact that nucleic acids exhibit a large number of double-helical conformational structures, which may be grouped into two classes, A and B; the A→B transition is induced experimentally by small changes in the relative humidity at which the macromolecules are held. The Davydov solitons have been proposed to provide the dynamic mechanism responsible for these transitions (Del Giudice *et al.*, 1982).

In this connection it is worth noting that, within the framework of rigorous field theory, the close relationship between solitons and Bose–Einstein condensation has been known for some time (Matsumoto *et al.*, 1979; Mercaldo *et al.*, 1981). In particular, the soliton has been described in terms of condensation of quanta, i.e., it has been reconstructed in terms of vacuum expectation values of a quantum field. This adds considerable weight to our arguments.

From the above analogy between biosystems and He II, as well as the evidence just cited, we conjecture that solitons are stable, since order in momentum space requires it. Now, if order in all quantum liquids is precipitated by the same underlying physical mechanism, then He II ought to display solitonic behavior. In this system the static (time-independent) effect of Bose–Einstein condensation may be probed by neutrons. The dynamic (time-dependent) effect is proposed in this paper to arise from the presence of solitons, as we shall see.

3. THE HARTREE APPROXIMATION AND THE CHOICE OF POTENTIAL

In the Hartree approximation we have the time-dependent self-consistent field equation

$$i\hbar\frac{\partial}{\partial t}\eta(\mathbf{x}, t) = -\frac{\hbar^2}{2m}\frac{\partial^2}{\partial x^2}\eta(\mathbf{x}, t) + \Lambda\eta(\mathbf{x}, t) + \eta(\mathbf{x}, t)\int |\eta(\mathbf{y}, t)|^2 V(\mathbf{x}-\mathbf{y})\, d\mathbf{y} \tag{1}$$

where $\eta(\mathbf{x}, t)$ is the single-particle state normalized such that $\int |\eta|^2\, d\mathbf{x} = N$, where N is the total number of particles in the system, and Λ denotes the chemical potential. Equation (1) must be simplified if a solution is to be obtained. Various choices of potential are possible:

1. The purely repulsive delta-function interaction is a popular choice (Gross, 1963):

$$V(\mathbf{x}-\mathbf{y}) = V_0\, \delta(\mathbf{x}-\mathbf{y}) \tag{2}$$

However, this is appropriate only for phenomena with typical length dimensions larger than the range of the interparticle force.

2. A somewhat more realistic potential is (Liu and Zamik, 1984)

$$V(\mathbf{x}-\mathbf{y}) = -V_0 \exp(-r^2/a^2) + \tfrac{1}{6}t_3\rho(\mathbf{R})\, \delta(\mathbf{x}-\mathbf{y}) \tag{3}$$

where a is the range of the attractive part, $\rho \equiv |\eta|^2$ denotes the fluid density, and t_3 is some adjustable parameter; further,

$$R = \tfrac{1}{2}(\mathbf{x}+\mathbf{y})$$

and

$$\mathbf{r} = \mathbf{x}-\mathbf{y}$$

3. An intermediate case between the previous two potentials is the Skyrme type, in which the attractive Gaussian becomes (Skyrme, 1959)

$$-V_0 \exp(-r^2/a^2) \to -t_0\, \delta(\mathbf{r}) + \tfrac{1}{2}t_0 a^2[k^2\, \delta(\mathbf{r}) + \delta(\mathbf{r})k^2] \tag{4}$$

where

$$\mathbf{k} = -\tfrac{1}{2}i(\nabla_x - \nabla_y)$$

In the limit as $a \to 0$,

$$-V_0 \exp(-r^2/a^2) \to -a^3 V_0 \pi^{3/2}\, \delta(\mathbf{r})$$

In other words, the intermediate case becomes

$$V(\mathbf{x}-\mathbf{y}) = -a^3 V_0 \pi^{3/2}\, \delta(\mathbf{r}) + \tfrac{1}{6}t_3\rho(\mathbf{R})\, \delta(\mathbf{r}) \tag{5}$$

4. More realistic potentials are known, examples of which are the HFDHE2 (Aziz *et al.*, 1979) and the HFIMD (Feltgen *et al.*, 1982) potentials.

In order to approach the superfluid problem a compromise must be struck among potentials 1-4. The work relating to solitons in the NLSE has so far been confined to potential 1, i.e., equation (2) (Nonnenmacher and Nonnenmacher, 1983; Segur, 1976).

Now, case (i) in Section 1 is not germane to our physical system, since the repulsive part of the interaction definitely plays a significant role. Case (ii) in that section, on the other hand, is inconvenient since the asymptotic behavior of the potential itself is quite specific and depends on the choice of potential we have in mind.

We proceed to investigate solutions to wave propagation in our weakly interacting Bose system using the potential given by equation (5). This is dictated by our desire to obtain a reasonably realistic, yet still analytically manageable, solution to the field equation (1).

4. THE EXACT SOLUTION OF THE FIELD EQUATION

4.1. On the "TCP" Solution

From the above arguments it follows that it is desirable to solve equation (1) with at least a rudimentary interaction potential mimicking the gross features of a potential that has both attractive and repulsive parts. Several progressive refinements of analytic detail may be brought into the description, but we start with a simplified form of the potential given by equation (5), in which the repulsive part V_R is taken to be density-independent:

$$V_R = \tfrac{1}{6}\tilde{t}_3\, \delta(\mathbf{r}) \tag{6}$$

We first note that exact solutions to the one-dimensional, time-dependent Landau-Ginzburg equations have been obtained recently (Tuszynski *et al.*, 1984a). These will be referred to as the TCP solutions. It should be emphasized, however, that this is essentially a one-dimensional problem; the three-dimensional counterpart is not amenable to any solution at present.

The order parameter

$$\eta = |\eta| \exp(i\phi) \tag{7}$$

has been expressed by TCP as a complex function dependent on both a space and a time variable. The first equation studied by them is

$$-D\eta_{xx} + [2A_2 + (n+2)A_{n+2}|\eta|^n + (2n+2)A_{2n+2}|\eta|^{2n}]\eta = -i\eta_t \tag{8}$$

where the subscripts denote differentiation with respect to the variables indicated.

It is importnat to remark at this stage that the phase Φ, which determines the quantum velocity of the soliton, as we shall see, is related to η; the one quantity implies the other. This is consistent with the well-known property of solitons that the velocity is proportional to the amplitude.

Defining the parameters

$$c_1 \equiv \frac{1}{D}\left(2A_2 + \frac{v^2}{2D}\right), \qquad c_2 \equiv \frac{2A_{n+2}}{D}, \qquad c_3 \equiv \frac{2A_{2n+2}}{D}$$

where

$$v \equiv -2D\phi_x$$

denotes the quantum velocity, and

$$\Delta \equiv 4c_3c_1 - c_2^2$$

TCP found the following solution:

$$\eta(x, t) = \frac{1+\Delta}{4c_1}\left\{\sinh[X_4 \pm nc_1^{1/2}(x-vt)\right.$$
$$\left. + \frac{1-\Delta}{4c_1}\cosh[X_4 \pm nc_1^{1/2}(x-vt)] - \frac{c_2}{2c_1}\right\}^{-1/n} \tag{9}$$

provided $c_1 > 0$ and Δ is arbitrary. Here

$$X_4 = \ln\{2[c_1(c_1\eta_0^{-2n} + c_2\eta_0^{-n} + c_3)]^{1/2} + 2c_1\eta_0^{-n} + c_2\} \tag{10}$$

where $\eta_0 \equiv \eta(x = vt)$. It is interesting to note that the quantum velocity v is nothing other than the superfluid velocity arising in the Madelung-Bohm transformation (Madelung, 1927; Bohm, 1952).

4.2. Discussion of the Case $n = 1$

Our attention is at present confined to equation (8) with $n = 1$. It follows that the restrictions for even n listed by TCP do not apply here. Further, in this case, the term $A_{n+2} = A_3$ will arise. However, this would be dropped in the Landau-Ginzburg free energy expansion, which lies at the root of the method of solution for equation (8), since only A_2, A_4, and A_6 occur in that expansion (Tuszynski *et al.*, 1984a).

Thus, for our weakly interacting Bose gas, taking

$$D \equiv -(2m)^{-1}; \qquad 2A_2 \equiv \Lambda; \qquad 4A_4 \equiv (-a^3V_0\pi^{3/2} + \tilde{t}_3/6) \equiv g$$

where m denotes the ^{4}He atomic mass, we find

$$c_2 = 0 \tag{11a}$$

$$c_1 = -2m(\Lambda - mv^2) \tag{11b}$$

$$c_3 = -mg \tag{11c}$$

$$\Delta = 8m^2 g(\Lambda - mv^2) \tag{11d}$$

$$g = -a^3 V_0 \pi^{3/2} + \tilde{t}_3/6 \tag{11e}$$

The condition $c_1 > 0$ becomes

$$v^2 > \Lambda/m \tag{12}$$

The solution (9), along with the parameters given by equations (11a)-(11e) and the restriction (12), represents waves propagating through the system with a constant velocity v ($>\Lambda/m$) and a permanent profile. It is crucial to note that, under these circumstances, the quantum velocity must exceed a certain critical value $v_c \equiv (\Lambda/m)^{1/2}$.

4.3. On the Choice of the Interaction Potential

More detailed analytic solutions of an exact nature may be followed up by a more careful consideration of the attractive part of the potential, for instance, by returning to the Gaussian shape [cf. equation (3)]. Alternatively, the analysis of Moszkowski and Scott (1960) may be attempted, in which the repulsion effectively cancels a part, but not all, of the attraction. The assumption here is that we may throw away the repulsion *and* the part of the attraction that is canceled. We may then work only with the remaining, relatively weak interaction.

4.4. Experimental Search for the Propagating Solitons

We believe that the essential qualitative features of this interesting phenomenon of wave propagation are already built into solution (11). Experiments should display this behavior, solitons being detected by their familiar properties (Kodomtsev and Karpan, 1971).

In this connection, restricted dimensions also come to mind, as in the original suggestion of He II solitons (Huberman, 1978). Even more appropriately, experiments could be carried out on ^{4}He adsorbed on porous Vycor glass, since this seems to behave as a dilute Bose system (Repply, 1984). Notwithstanding the underlying difficulties, it is worth pursuing this matter in view of the fundamental importance outlined in the following section.

5. CONCLUSIONS

We have been motivated in this work by two different factors:

1. The recent deep understanding of living matter, which is capable of achieving a steady state away from thermal equilibrium through the appropriate use of both metabolic energy and soliton propagation in the presence of a single quantum mode macroscopically occupied by phonons.

2. The experimental discovery of a physically realizable, weakly interacting Bose system (Reppy, 1984), which is amenable to a quantum mechanical treatment gradually approaching a reasonable microscopic description (Ghassib and Khudeir, 1986; Chela-Flores and Ghassib, 1986).

By considering a generalized Madelung fluid, we have inferred that propagating solitons should exist in the weakly interacting Bose system and perhaps even in He II itself. The analogy with biophysical systems has led to the remarkable conclusion that these solitons play the vital role of propagating Bose-Einstein condensation through the medium, thereby effecting "phase-locking"—the key mechanism in the phenomenon of superfluidity (Anderson, 1966). It is not unreasonable, therefore, to suggest that a Bose-Einstein condensate might become a superfluid through the mediation of solitons.

Further work should shed more light on the fundamental questions involved here, including the suggestive remarks that soliton solutions for the Madelung fluid are represented by poles occurring in the scattering data in the complex energy plane (Nonnenmacher and Nonnenmacher, 1983)—just where bound states (dimers) occur. This raises the important question: What is the relation between solitons and dimers?

While a conclusive answer requires more rigorous work, it is already clear that these are indeed related. In particular, only those operators of the NLSE that give discrete spectra yield solitonic solutions. For continuous spectra, decaying oscillations are obtained, as we have seen. Since the most highly acclaimed He-He potentials sustain a diatomic weakly bound state—a dimer (Ghassib, 1984)—it is reasonable to anticipate that, when used in the above framework, these should yield solitons as well.

There is no evidence to suggest that this relation is causal; however, the mere fact that both dimers and solitons appear, if at all, simultaneously is intriguing enough to merit further investigation.

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